Mechanics Examples Sheet 4 — Solutions

1. use the formula

constant acceleration

$$V = u + at$$

final velocity, initial velocity

$V = 0$, as particle finally comes to rest.

$a < 0$.

$$0 = u - |a|t \Rightarrow \underline{\underline{t = \frac{u}{|a|}}}$$

The distance can be obtained using the formula

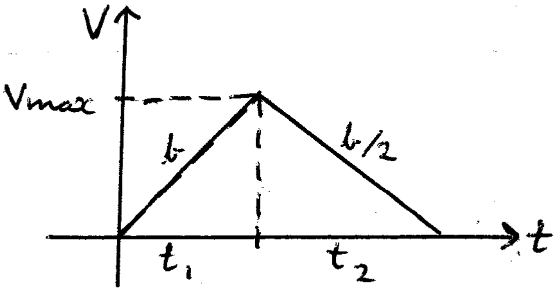
$$V^2 = u^2 + 2aS.$$

$$0 = u^2 - 2|a|S \Rightarrow \underline{\underline{S = \frac{u^2}{2|a|}}}$$

N.B. the result could also have obtained on using the formula

$$S = ut + \tfrac{1}{2}at^2.$$

2. The velocity-time graph is



The gradients are straight lines because the accelerations are constant.

Notice that the area under the graph represents the total distance moved by the particle: $S = \int v\,dt$.

The total distance covered in time t is

$$d = d_{t_1} + d_{t_2}$$

$$= \frac{1}{2}V_{max}t_1 + \frac{1}{2}V_{max}t_2, \quad t = t_1 + t_2$$

$$= \frac{1}{2}V_{max}t. \qquad -(1)$$

Now, as accelerations are represented by the gradients in the velocity-time graph, we have

$$b = \frac{V_{max}}{t_1} \quad , \quad \frac{b}{2} = -\frac{V_{max}}{t_2} .$$

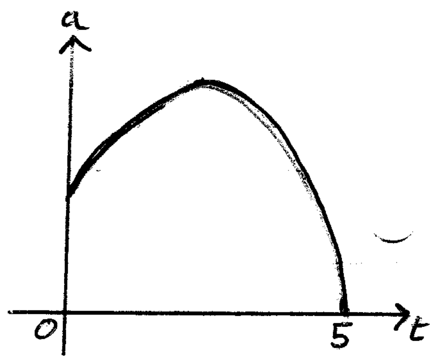
Hence,

$$V_{max} = \frac{b}{3}(t_1 + t_2) = \frac{b}{3}t .$$

substituting into (1):

$$d = \frac{1}{6} b t^2$$

3. N.B. The standard formulae cannot be used here because the acceleration is NOT constant.



$$a = 5 + 4t - t^2$$

(i) For maximum acceleration:

$$\frac{da}{dt} = 0 \Rightarrow 4 - 2t = 0 \Rightarrow t = 2$$

$$\therefore a = 5 + 4(2) - 2^2 = 9$$

(ii) The greatest speed occurs when $a = 0$ i.e. at $t = 5$ ($t = -1$ is not within the range.)

$$\therefore \text{speed} = \int_0^t a\,dt = \int_0^5 (5 + 4t - t^2)\,dt = \frac{100}{3}$$

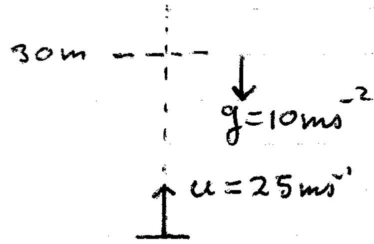
(iii) $u = \int_0^t a\,dt' = 5t + 2t^2 - \frac{t^3}{3}$, where u is speed.

$$\therefore \text{distance}, S = \int_0^5 \left(5t + 2t^2 - \frac{t^3}{3}\right) dt$$

$$= \frac{1125}{12}$$

$$= \frac{375}{4}$$

4. use $S = ut + \frac{1}{2}at^2$,



$u = 25\ ms^{-1}$
$a = -g = -10\ ms^{-2}$
$S = 30\ m$.

Substituting values into eqn:

$$t^2 - 5t + 6 = 0 \quad \Rightarrow (t-2)(t-3) = 0 \quad \Rightarrow t = 2, 3.$$

Hence, the time for which the particle remains above a height of 30 m is

$$\underline{\underline{t = 3 - 2 = 1\ s}}$$

5. $$\underset{\sim}{a} = \frac{d}{dt}\underset{\sim}{v}$$

$$\Rightarrow \quad \underset{\sim}{a} \cdot \underset{\sim}{v} = \underset{\sim}{v} \cdot \frac{d}{dt}\underset{\sim}{v}, \qquad \text{on dotting with } \underset{\sim}{v}$$

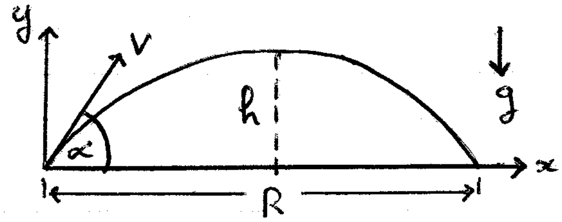
$$\Rightarrow \quad \frac{d}{dt}(\underset{\sim}{a} \cdot \underset{\sim}{x}) = \frac{1}{2}\frac{d}{dt}(\underset{\sim}{v} \cdot \underset{\sim}{v})$$

$$\Rightarrow \quad 2\underset{\sim}{a} \cdot \underset{\sim}{x} = v^2 + c, \quad \text{on integrating } (c = \text{const.})$$

Applying initial conditions gives $c = -u^2$. Hence,

$$v^2 - u^2 = 2\underset{\sim}{a} \cdot \underset{\sim}{x}$$

6. use $S = ut + \frac{1}{2}at^2$



x: $x = V\cos\alpha\, t$ — (1)

y: $y = V\sin\alpha\, t - \frac{1}{2}gt^2$ — (2)

$$\therefore \frac{dy}{dt} = V\sin\alpha - gt$$

Maximum height, h, occurs when $dy/dt = 0$, i.e.

$$V\sin\alpha - gt = 0 \quad \Rightarrow \quad t = \frac{V\sin\alpha}{g}$$

substituting this into (2) yields

$$\underline{\underline{h = \frac{V^2\sin^2\alpha}{2g}}}$$

The range of the particle, R, is the maximum horizontal distance achieved. This occurs at $y=0$. From (2)

$$0 = V \sin\alpha \, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \frac{2V\sin\alpha}{g}$$

Substituting this into (1) yields

$$R = (V\cos\alpha)\left(\frac{2V\sin\alpha}{g}\right) = \underline{\underline{\frac{V^2\sin 2\alpha}{g}}}$$

R is max. when $\sin 2\alpha = 1 \Rightarrow \underline{\underline{\alpha = \frac{\pi}{4}}}$